AE3030 NOTES Test 3

Compressible Potential Flow Equation

Limiting ourselves to 2D flow (this is page 48)

Velocity Is the gradient of the porential

V = gradient(phi)

U = diphi/dx

V = dphi/dy

Lopooking at the continuity equation, we have

Dp/dt (which goes to zero) + drho*u/dx + drhov/dy =0

We can’t pull rho out because we can’t assume incompressible.

Product rule in that case:

Rhodu/dx + u drho/dx + rhodv/dy + vdrho/dy =0

U = dpjhi?dx and v = dphi/dy

So we have, if you substitute the potential flow equations back in….

Rhod^phi/dx^2 + dphi/dx * drho/dx + rho*d^phi/dy^2 + dphi/dy * drho/dy = 0

We get a (page 49)

We want all our equations in terms of phi

Recall Euler's equation:

dP = -rhoVdV

Assumptions:

Steady, inviscid, no body forces, valid ofr compressible, a stream tube

Derivation from Euler';s equation starts page 49For isentropic flow….

P = C_p^gamma

Gamma = C

If you set 1 and 2 equal….you get the bottom of page 50

The ,w which is on Page 52, is the Velocity Potential Equation

It's a second Order PDE, highly non-linear, and a superposition of sources and sinks will NOT work to model it.

For incompressible flow, a ___> infinity

All you're gonna be left with is Laplace's equation

For 3D flow ----- see slide 13 AE3030_Lecture II

Soooo we can't really solve this eqatuoin.

We ned to linearlze it.

Linearization!!!

In porder to linears the velocity potential equation, we need to make a few assumptionsL:

Bodies being analysed are thin.

Low thickness to chord ratio

-Have a mild camber

The freestaream velocity is to be disturbed only slightly.

SUBSONIC AND SUPERSONIC ONL:Y

No transonic or hypersonic.

Subsonic – incompressible

Supersonig – everywhere M >1

Transonic – freestream is less than M =1, mach M>1 or M= 1 on body

Hypersonic = temperature effects take over compressiblility effects

OK time to linearise!!!

There is a picture of an airfoil and perturbation velocityies….. so…… (page 53)

Assume the potential is now phi = v_infx + phi_hat , which is the perturbation

So we have u = dphi/dx = v_inf + dphi/dx (which is u_hat)

Etc.

So you can assume that the second derivatives of the phi function with respect to x and y are all equal to the perturbation second drerivatives.If you substitute that stuff into the velocity potential equation, you get the top of page 54.

If you look at the second order terms….

You can reason that they are all negligible. (Proof on page 54)

Let's take a look at the nergy equation from lecture 1….

CpT + v^2/2 = CpT_inf + v^_inf^2 / 2 (This is page 55

You can do some derivationand relations……

SO in the end you are left with this equation on the top of page 57

Called the SMALL DISTURBANCE EQUATION

It's the governing equation for compressible subsonic and supersonic flow.

This equation is linearized and non-exact! It's approximate.

Sp……look at the pressure coefficient

Pressure coefficient

$C_p = P - P_{inf} / q_{inf}$

Derivation without and assumptions made:

$C_p = 2/\gamma M_{inf}^2 [P/P_{inf} - 1]$

Let's consider adiabatic, calorically flow

(btw no assumoptions were made for the top equation – it was exact)

You get a P/P_{inf} equation at the bottom of page 58.

There is another equation for exact Cp, but assuming isentropic (neglecting shocks). It's page 60.

You can do some more manupuulation to come up with

$C_p = -2 \hat{u} / V_{inf}$

Wall boundary condition

We know that velocity can only be tangential and not normal to the surface.

…..

You get that phi_hat is about V_inf times df(x)/dx (which is the local body slope.

SUMMARY OF LINEARIZATION OF COMPRESSIBLE FLOWS:

Governing equation: (1-M_inf^2) * phi_xx + phi_xx = 0//

Boundary condition: phi_hat_y = V_inf * df(x)/.dx

Cp = -2phi_hatx / V_inf

These are all on age 63)

At the end of the world wars, they were looking for easy solutions for transonic regimes.

PThe Prandtyl-Glaurent Compressiblity Correction

(WHICH IS NOT VALID FOR TRANSONIC FLOWS)

Want something similar to incompressible eq…..Laplacian..

So…. Let xi = x

Eta = Beta y

Beta = sqrt(1-M_inf^2)

So we have now

B^2Phi_xx + phi_yy = 0

(those phis are phi_hats)

The new velocity potential function is phi_bar(xi,eta) = beta*phi-hat(x,y)

Some derivation starts on page 63……In transformed space, this problem becomes an incompressible problem!!

The shape of an airfoil in transformed space is the same

Y = f(x)

Eta = q(xi)

Proof of similarity of space is on page 65

:The pressure coefficient has changed a little in the alteration.

Cp = Cp, o / beta

This is Prandtl-Glauret's Compressibility Correction!

(Page 76)

Keep in mind: This is not to be used fortransonic or hypersonic

Still assuming steady

Irrotational

And no vorticity

Isentropic flow

Reversible adiabatic

Still assuming inviscid flow.

(BL regin is computed separately)

-Flow away from walls is considered inviscid

ALSO

It's used for a particular airfoil and for a particular angle of attack.

Because we are assuming inviscid, subsonic, compressible flow… Cd = 0.

This is d'Alembert's paradox.

Anything less than M = 0.3, then you can assume incompressible.

But from 0.3 to 0.8, this stuff is valid.

There are MORE, better improved compressibility (similarity) equations

Karmen-Tsien Equation, etc. These are in slides (but a little incorrect! Best to double-check)

TRANSONIC REGIME

Critical Mach Number

This is the mach number of the freestream when the flow over the body first gets to M = 1.

There are good diagrams on page 69

When Mach number is greater than M_cr, there is a sonic line that develops over the upper surface of the airfoil.

Doesn't necessarily form a shockwave, though.

What happens to the boundary layer>

When there is a shockwave, the static pressure increases a crazy amount, and we get boundary layer separation.

There is massive separation fdrag.

Votcies, a lot of rotationality,

Varying pressure,

Unsteady flow, periodic buffeting.

Coefficient of lift in 4 difference situations….

(Thieree is a fgreat graph on page 69)

You can drive the shockwave back to the trailing edge

You only get Bow shocks forblunt edged bodies.

If you have a sharp leading edge….you get oblique shocks.,

Mach tuck is caused by lift being generated in the aft regions of the airfoil.

As Cp moves backwards, pitching down moment is unrecoverable.

So…

Review from last time:

M_inf as it approaches Mcr, depending on AoA and geometry….

The AoA and geom will determine how much flpow speeds up. The faster it is over the suction peak, the sooner you will reach Mcr.

Let’s say suction peak is A…..

(Some derivations on page 73)

There is the equation on how to find Mcr on page 74

THIS IS UNSOLVEABLE BY HAND!!)

Transonic flow

This is the end of the math, because this stuff is non-linear.

As freestream mach number increase, you hit M = 1 at the suction peack.

What is transonic flow?

-Flow in which significant regions of subsonic and supersonic flows exist together.

(Freestream M could be over 1)

M_inf < 1 …. In this case shockwaves can happen on the airfoil

.

If M_inf > 1, then there may be a bow shock, and a subsonic regime after that, then another supersonic regime with some oblique schocks at the trailing edge. (picture on page 75)

Pressure conditions at the end determine whether there are sub or super sonic solutions to oblique shocks.

There is a picture on page 76 of expansion wvaves….

Multiple expansion waves and compression waves coalesce from a shock

If you keep increasing the Mach number, shock ravels towards the trailing edge.

Kutta condition and pressure_inf must be satisfied.

Shocks turn the flow to meet the Kutta condition.

-Velocities have to match

P_static also has to match

If the top and bottom shocks are of different strengths, they have different entropies…..this causes a slipline.

What is a slipline?

Discontinuity for entropy., temperature, ad rho

BUT

P and v are discontinuous.

Let's look at a few Cp lots.!!!

These are on page 77.

Wave drag

What is wave drag? Why do shocks cause it?

You have a difference in velocities….. because the shocks make entropy go up, which means that the density after a shock is lower than rho_inf.

There is a deficit in velocity because of the entropy jump.

When you have lower velocity at the TE, then not as much mass gets transported through.

When you have oblique shocks at the TE, wave drag is asmaller

The BL is attached to the airfoil, so you have smaller separation drag.

If you have a strong enough engine to combat the sound barrier, then you’re good!

Commercial transport:

IncrASING RANGE IS IMPORTANT

In order to maximize range:R

$dR = V_{inf}*dt$

dW is the change in aircraft weight

T is thrust

$dW = T*sfc*dt$

(specific fuel consuptiom

$dR = V_{inf}dE / T(sfc) * (W/W)$

Thrust = Drag and W = L for steady, level flight

$dR = V_{inf} * L * dW / sfc * D * W$

Do the integral….

R = V/inf / sfc * Cl/Cd * ln (W_takeoff / W_landing) This is the Brequet Range Equation. This is on page 80.

So….as aerodynamicists, how to maximize Cl/Cd?

Cl/Cd has the highest ratio at just after Mcr. This is why cruising is at M = 0.7 or M = 0.8.

How to reduce Wave Drag:

3 major design consideration:s

I) Supercritical airfoils
II) Ii) wing-body interactions and "areqa rule"
III) Iii) swept wings at transonic speeds)./

*** supercritical airfoils

So…if you look at the NACA 00012…..it's got a blunt edge but Whitcomb said design the top to be flat

With a cusp towards the TE to recover lifty.

The LE should be blunter to get to M = 1 faster.

In order to not speed up flow on top = flat

-blunt nose to increase to M = 1 gfaster

Cusp to recover lift

-thicker bottom for structural and fuel concerns

ii) Area rule

Whitcomb noted that indenting the body of a wing-body combination so that the combination has nearly the same axial distribution of cross-sectional area as the original body along would result in the transonic drag rise.

Whitcomb's area rule ^^^^

Sharp disonctinuities in area cause wave drag

Change in fuselage shape to mmake the crosssectional area change more smooth…. You can prove this mathematically

(some graphs on page 82/83)

iii) Wing sweep

Basically you;'re tricking the wing into thinking that V_inf is slower with swing sweep.

Effective veloicity is Mach #....reduced to the cosine of the angle related to the swept wing.

Infinite aspect ratio assumption

-no spanswise derivatives --- all y-derivatives = 0

(There are slides of governing equations for all this, but I stopped taking notes somewhere…..this iwws page 83)

The benefits and drawbacks of a swept back wing….

(good pictures on page 84)Boundary Layer fences are sometimes used to obstruct spanwise flow and keep wings from stalling

Boundary layer fences aren't really used anymore though….vortex generators are still used. They are placed at angles.

Swept forward wings…..

Tip stall is avoided

The fuselage acts as a boundary layer fence, reducing root stall.

You get high lift, but the aeroelastic effects have a tendency to cause wings to snap off!!

Subsonic flow over Finite wings

Governing equation:

$B^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = 0$

(all phis are phi-hats)

The explanation for this is page 85-86.

Basically you ge the same similarity transformation equation for Cp, but you also have to transform the wing.

ONTO SUPERSONIC FLOW

So, the supersonic regime is when $M_{inf} > 1$

Pressure waves propagate at the speed of sound to let the other air know if its presence / disturbance.

For subsonic flows, disturbance causes progrogation at the speed a*t.

Definition of mach angle (page 86)

Linearized 2D Supersonic Flow Potential

Previously: se had $(1- M_{inf}^2)(d^2\hat{\phi}/dx^2 + d^2\hat{\phi}/dy^2 = 0$

This came from continuity.

-Then we linearized it and solved for a small case

-Etc etc…

For subsonic flow, $1-M_{inf}^2 > 0$

This gives you an elliptic PDE

For Supersonic flow: $1-M_{inf}^2 < 0$

Which is a hyperbolic PDE

Let’s multiply by -1.

1- M^2 (phixxx – phyy = 0

Lamda = sqrt(Minf^2 – 1)

Phi_hat = f(x – lamday) is a solution. HOW?

This derivation is on pages 87 and page 88.

The problem is that the solution is not specific. What is f? it could be anything.

Lamda^2 f’’ – lamda^2’’ = 0

Derive derive…..math math….

Page 89

All disturbances created at the wall (phi_hat) propagate unchanged away from the wall long Mach lines.

The slope downstream above the wall:

No disturbance.

If there is a disturbance….the disturbance propogates up the Mach line

Second solution

There’s another solution, guys!!!!! Omg

The second solution is for the bottom.

However!!!

The discontinuous solutions are discontinuous.

Therevofore interactions are NOT possible.

Freestream ca't interact with the Mach lines.

Kutta condition is still satisfied.

Chocks at trailing edge.

Cp…… remember….

Cp = -2u-hate/V_inf.

Basically Cp is proportional to 1 / sqrt(M^2 – 1)

(Ackert's Rule)

Because it's linearized, this equation blows up at M_inf = 1.

Lower surface Boundary condition….

Math math…

ACKERT'S RULE!!!

Page 92 and 93.

Basically camber is useless for supersonic flows

Cl is dependent on alpha and M_inf only.

Cp = about 2/lamda [dY/dx]

ACKER'T

ACKERT'S RULE

Better look at page 93 and onward for Cl and Cd for a flat plate

There is an example of Ackert's rule calculations on page 97.

And also page 98.

3-D Supersonic flow

-aerodynamics involves interactions between wing, fuselage, enginesm and empanage (back region)

-CFD has evolved to a point where analysts and designers increasingle rely on calculations, complemented by costly wind tunnel and flight test data.

Linearized potential flow analyses are nevertheless useful..

SOME USEFUL RESOURCES AOBUTH IHIS ARE NASA CONTRACT REPORTS.

Drag is a sum of…

-skin friction drag

-induced drag (wing and tails)

Due to lift, proportional to Cl^2

-Wave drag

-trim drag

Even if zero lift is being generated, you still have wave drag.

Skin Friction Drag

-Based on flat plate theory for turbulent flows, divide surface into patches of area dletaS

-on each patch, we compute the area and skin friction coeff Cf and associated drag.

Sump up the Cf over all the patches to get total

Calculate wall temperature

Find equativalent imcompressible flow temp T'

Look up viscosity mu' at T' using Sutherlands's Law

-Find incompressible density rho' from equation of static rho' = rho_inf(T_inf/T')

Induced drag'

-caused by rearward rotation of L' due to downwash as incompressible flow.

-An empirical expression may be fond for each lifting surface.

Can use panel methods

Wave drag

You can see the effects of wave drag with a Cp vs. Surface area lots

When Cp is high, you can expect a shock.

Mach Cone……

There a cones of silence (picture on page 100)

Supersonic and Subsonic LE

(Pictures on page 101

Subsonic LW has the advantage of haining no shocks on the body.

BUT if you sweep the wings too far, you get:

-a large pitching moment

-structural problems

Positive: There is no interaction between upper and lower surface for supersonic flow

In practice

-Most wings have a supersonic trailing edge

-designers strive to have a subsonic LE.

Concorde moves its fuel around during supersonic flights.

Wave drag: area rule

BUT you want to maximize volume because you want the maximum payoad.

Instroduction to Hypersonic Flow

Hypersonic aerodynamics is a very broad topic

-can't clearly say that there is a line between sonic and hypersonic

BL – space between shock wave and body – thin and rotational.

For compressible BL, temp effects lower mass flow rate

-Shock layer

-the gasses may be dissociated or even ionized

Hypersonic flow "features"

-→ High -Temp effects

-M = 20 or 30….kinetic energy turned into internal energy

-Gassses have….3 translational and 2 rotational modes

At high mach numbers, you get more modes – vibrational modes

And gas dissociation

-Youget Cp as a fxn of pressure and temp

-Chemical reacionts

High temp effects also make flow non-equillibrium

(reference timescale is longer than the chemical reaction timescale

High altitude effects:: low-desnity

-continuum breaks down

-velocity and temeprpature slip

Shockwave very close to the body

-genreatrtes an entropy later, interacts with the boundaery layer.

-everything is viscous, everything is rotational

-entropy layer is a gradient in entropy

Some more high temp effects

Equations on page 103

*I***

Prandtl Number - it's the Ratio of Viscous diffusion rates to Thermal Diffucsion rates

Nu = kinematic viscosity = mu / rho which is viscous diffusivity.

Here is a new constant for you:

Alpha = k / rho * Cp. K is thermal conductivity.

This is thermal diffusivity.

Pr = nu ./ alpha. Which is the equivalent of mu * Cp / k. (This is page 104)

The flow temperauter in the viscous-shock layer is extremely high!!

Specifical heats Cp and Cv and their rtio gamma are fcn of temp b/c new degrees of freedom arise

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We are trying to get a simple estimate of Cp, Cl, Cd, and Cm.

-Hypersonic flow is highly non-linear though!

Newton said…that air travels in a straight lline.

When it hits a wall, it loses velocity and slides against the wall. The normal energy becomes the Pressure Force.

(a derivation on page 105)

We have a re-entry vehicle (diagram page 105)

In 3-D, $C_p = 2 \, \text{abs}(v_{inf} * n)^2 / V_{inf}^2$

THIS IS VERY WRONG FOR SUB, TRANS, and SUPER SONIC FLOW.

But it’s more valid for hypersonic flow

There are fewr collisions because of rarified air

-The shock sits so close to the surface , it may as well be hitting the surface

---

Newton’s theory modified….

Cp is overestimated, so let’s replace somethings..

C_{pmax} is now $C_{pmax} * \sin^2(\theta)$

Cl and Cd are on page 106.

I didn’t do the derivation so you have to look at the slides if you want to see how that was done)

Derivation of Energy Equation Viscous Flow

This is all on page 107 to 108.

And it’s

…it all becomes Sutherland’s Law

Sutherland’s Law:

(an empirical curve fit)

Describes how viscosity varies with temperature.

Mu/mu_o = (t/To)^3/2 ((To + S1) / T + S1)) S1 is 110 Kelvin for air.

YEAH

That’s it.

This class is over!!!!!